International Business Cycle Accounting

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Abstract

I have developed a method that can provide insights to researchers to better specify their quantitative models in international business cycle studies. The guidance comes from the application of an accounting procedure based on a prototype model of international growth that includes wedges capturing all the potential frictions and distortions of markets. For each country, I include an efficiency wedge, labor wedge, investment wedge, government wedge, preference wedge, and foreign asset wedge. I then demonstrate the method by applying it to the US and Canada during the Great Recession (2007-2008). I found that the economic downturns in both countries during this period were primarily due to the US investment wedge, US labor wedge, and US efficiency wedge, with the Canada investment wedge playing a secondary role. These results suggest that the crisis originated in the US and was propagated to Canada.

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1 Introduction

The synchronization of business cycles across countries is a well-known fact. However, the literature on the international business cycle has shown that many models hardly replicate this fact. It has then been documented in international macroeconomics a lot of puzzles[1]. To solve these puzzles, economics researchers build detailed models in which they add frictions to replicate economic fluctuations observed in data. However, they face significant difficulties with which frictions and where to introduce them in the model. In this study, I propose a method that could facilitate those choices, and I apply the method to the US and Canada during the Great Recession.

My method is an extension to the open economy of the *business cycle accounting* method proposed by Chari, Kehoe, and McGrattan (2007). The method has two components: a theoretical result and an accounting procedure. The theoretical result consists of building a prototype model in which we include time-varying wedges that distort the equilibrium of the economy (otherwise in perfectly competitive markets). This prototype model generalizes a large class of detailed models with frictions. In other words, each detailed model would be equivalent to the prototype model with some specific wedges. My prototype model is built on a canonical two countries growth model (as in Backus, Kehoe, and Kydland (1992)) in which I include some wedges. I consider six wedges for each country in my framework and label them *efficiency wedge*, *labor wedge*, *investment wedge*, *government wedge*, *preference wedge*, and *foreign asset wedge*. Those wedges intend to capture the frictions in each country as well as those resulting from the exchange relation between the countries. Thus the labor market frictions, the financial market frictions, the trade frictions, etc. will be captured by one or a combination of those wedges in the model.

The accounting procedure consists of measuring, first of all, the wedges. Those wedges capture how much the realized allocations are distorted from the competitive equilibrium allocations. For that purpose, we use the data and the equilibrium conditions of the prototype model. To express those wedges in a meaningful way, in the second step, we evaluate the contribution of the wedges. This means that we feed the values of the wedges, one at a time or by combination, to the model. These experiments help assess how much the fluctuations of output, investment, consumption, and labor are due to wedges considered separately or in combination. For example, if we want to assess the role of the labor wedge in the fluctuation

[1] Anomalies that occur when model predictions or results differ from the facts observed in data. (Obstfeld & Rogoff, 2000) has identified six puzzles.

of aggregate variables we feedback in the model that wedge and keep constant all the remaining wedges.

The accounting procedure can be viewed as quantitative application of the theoretical result. I use the method developed to study the business cycle relation between the US and Canada during the Great Recession of 2007-2008. My goal from this exercise is to assess the channel through which the two economies were related during this crisis. I find that the US labor wedge, US efficiency wedge, and US investment wedge explain the major fall in output and labor in Canada during the recession. The decline in investment in Canada, on the other hand, was caused by the investment wedge of Canada. The US economic downturns during the recession were caused mainly by the US wedges. More specifically, the decline in US output and investment was due to the US labor wedge, and the US investment wedge. On the other hand, the decline in US employment was induced by the investment wedge of Canada combined with the US investment wedge. Those results can be compared to business accounting in the closed economy as performed by Brinca, Chari, Kehoe, and McGrattan (2016). While in a closed economy, all the aggregate fluctuations are imputed to frictions in the home country, my approach shows that the main causes could come from abroad. That is the case from the fact that the main wedges that explain the decline of Canada's output, during the recession, were the US efficiency wedge and the US labor wedge.

It was documented that the 2007-2008 Great Recession was a financial crisis[2]. My findings suggest then that the financial frictions manifest themselves not only as investment wedges but also as efficiency wedges and labor wedges. The US investment wedge, US labor wedge, and the US efficiency wedge account mainly for the downturns in both US and Canada. As my findings indicate that the frictions in the US economy explain the fluctuations in Canada's aggregate variables I infer that the great recession originates in the US and then propagates to Canada. The trade relationship as well as the financial transactions between the two countries are probably the channel of this transmission of the business cycle. Indeed, the prominent role of the US investment wedge in the decline of Canadian investment during this period suggests that the financial shock that occurred in the US has created some investment distortions in Canada.

The International Business Cycle Accounting method can help address different questions involving the relationship between countries. For instance, it can provide insights into the extent to which shocks to the US labor market or the US financial market can affect the economic condition in Canada or vice-versa. This paper is then related to the literature on the causes of business cycle synchroniza-

[2] Bordo (2012) shows that it is the financial crisis that lead to the recession

tion across countries. The first driven source of the business cycle synchronization explored was the productivity shocks (Backus et al. (1992), Heathcote and Perri (2002)). Then some authors contribute to the literature by combining technology and non-technology shocks to explain the international business cycle (Stockman and Tesar (1995), Wen (2007)). Other authors have investigated the role of input linkages and production networks in international business cycle synchronization (e.g. Kose and Yi (2006), Bems, Johnson, and Yi (2010), Johnson (2014), Eaton, Kortum, Neiman, and Romalis (2016) and Huo, Levchenko, and Pandalai-Nayar (2023)). Nevertheless, the identified source included in the model explained a fraction of the business cycle across countries and/or generated predictions inconsistent with the data known as *puzzles*. Many papers have investigated such anomalies in international macroeconomics such as Obstfeld and Rogoff (2000), Kose and Yi (2001), Kose, Otrok, and Whiteman (2003) and Ambler, Cardia, and Zimmermann (2004)). My paper, instead of considering a particular drive source, contributes to this literature by investigating all the possible sources that explain the international comovement observed in data. Thus, all the frictions together in the model will explain, by construction, the 100% of the business cycle synchronization. Therefore, my methodology intends to shed light on the source of friction that explain business cycle synchronization across countries. It is a diagnostic tool for the business cycles of a particular economy in its relationship with other countries. It acts as a prism for light. It distinguishes not only the shocks that affect the fluctuations of the aggregate variable but also the origin of those shocks (in terms of home shocks or foreign shocks).

Regarding methodology, the most closely related work is the seminal paper of Chari et al. (2007). A number of papers were dedicated to applying the methodology to study business cycles for many countries such as OECD[3] countries, China, Japan, and some developing countries (e.g. Kobayashi and Inaba (2006), Lama (2005), Chari et al. (2007), Gao (2007), andBrinca et al. (2016)). Whereas the methodology in those papers is designed for a closed economy, I extend it to open multiple countries. Thus, I allow a description and a quantification of the relations between countries while (Chari et al., 2007) method summarize all the interactions of a country with the rest of the world into the government wedge. Otsu (2009) has also developed a version of international business cycle accounting and has applied it to study the business cycle correlation between Japan and the US during the 1980-2008 period.

After the current introductory section 1, the rest of the paper is organized as follows. In section 2 I present the benchmark prototype model. In section 3 I

[3]The Organization for Economic Cooperation and Development

describe the accounting procedure of my method. The section 4 is dedicated to the description of the application of the method and the findings. After that, I make a discussion around the results found in section 5. The section 6 summarizes my findings and suggests some directions for further work.

2 Description of the Benchmark Prototype Model

The model is a competitive version of a two-country growth model as in Backus et al. (1992) except that each country produces a single specific tradable final good. I introduce wedges in relevant markets that represent distortionary shocks. Each country i consists of a representative household, firm, and government. The model is set up as follows.

Final Good Firms.– The representative firm in each country produces the aggregate output y_t^i from local capital stock k_t^i and local labor l_t^i using the production technology $F(.)$:

$$y_t^i = A_t^i F(k_t^i, (1+\gamma)^t l_t^i), \tag{1}$$

where the aggregate TFP is composed of labor-augmented technical progress captured by the rate $1+\gamma$ (assumed constant) and the stationary component A_t^i.

The final good is specific to each country. Thus, in the country i, the aggregate output serves for the home household consumption, foreign household consumption, home investment, and home government consumption.

Finally, the profit maximization problem for the final good firm can be written as

$$\max_{k_t^i, l_t^i} \left(p_t^i A_t^i F(k_t^i, (1+\gamma)^t l_t^i) - w_t^i l_t^i - r_t^i k_t^i \right), \tag{2}$$

where p_t^i, w_t^i, and r_t^i are respectively the price of the goods, the wage rate, and the rental rate on local capital.

Households.– The households in each country maximize their expected lifetime utility (equation 3) over per capita home consumption good $c_{ht}^i(s^t)$, per capita foreign consumption good $c_{ft}^i(s^t)$ and per capita labor supply $l_t^i(s^t)$. s^t represents the current state of nature. As all the variables in the model depend on the state variable s^t, for convenience I can omit them. Let E_t be the expectation operator relative

to the probability that each state event occurs.

$$E_t\left[\sum_{t=0}^{\infty}\tilde{\beta}^{it}U(c^i_{ht},c^i_{ft},1-l^i_t)N^i_t\right], \tag{3}$$

where $\tilde{\beta}^i$ is the discount factor, and N^i_t is the population with a growth rate denoted $(1+\gamma^i_n)$.

The maximization problem of the households is subjected to the budget constraints

$$\begin{aligned} &p^i_t c^i_{ht} + p^j_t(1+\tau^i_{ct})c^i_{jt} + (1+\tau^i_{xt})p^i_t x^i_t + \frac{N^i_{t+1}}{N^i_t}b^i_{t+1} \\ =&(1-\tau^i_{lt})w^i_t l^i_t + r^i_t k^i_t + (1+r^*_t(1-\tau^i_{bt}))b^i_t + Tr^i_t, \end{aligned} \tag{4}$$

where for a variable Z^i_j the subscript j is the origin country and the subscript i is the destination country. x^i_t is per capita investment, b^i_t per capita non-contingent international claim [4], r^*_t is the world rate return on risk-free securities, and Tr^i_t is per capita lump sum transfer from the government. τ^i_{ct}, τ^i_{xt}, τ^i_{lt}, and τ^i_{bt} represent distortionary taxes on household foreign consumption goods, investment, labor income, and foreign asset respectively.

Investment is assumed to follow the capital law of motion:

$$\frac{N^i_{t+1}}{N^i_t}k^i_{t+1} = ((1-\delta)k^i_t + x^i_t), \tag{5}$$

where δ is the depreciation rate of capital.

International Asset Market.– The households of each country trade on the foreign asset market a one-period non-contingent asset. At period t a household can contract an asset b_{t+1} (maturing at period $(t+1)$ and pay back the existing asset b_t issued at period $(t-1)$ including the interest at the world interest rate r^*_t.

As the world economy is constituted of the two countries, in each period, since the asset is zero net supply at the world level, the international asset market clearance condition is

$$N^h_t b^h_t + N^f_t b^f_t = 0 \tag{6}$$

Government.– The government of each country collects taxes from households, purchases goods and services, and rebates the remaining to the household as a

[4] It is an asset that is due to the foreign country, or invest in foreign country

lump-sum transfer in order to satisfy its budget constraint:

$$g_t^i + Tr_t^i = \tau_{ct}^i p_t^j c_{jt}^i + \tau_{xt}^i p_t^i x_t^i + \tau_{lt}^i w_t^i l_t^i + \tau_{bt}^i r_t^* b_t^i \tag{7}$$

While the distortionary can affect the agents' decisions in an economy, thus affecting the business cycle, the scope of this paper is not to study them. The government taxes in this set capture all the shocks that can affect each market. They measure the wedges. As a result, I do not use data on taxes in this work.

Definition of the wedges.– The set-up of the model includes six wedges for each country capturing all the disturbances in the economy. For each country the six wedges are defined as follows:

1. The *efficiency wedge* $\Delta_{at} = A_t$ resembles a time-varying technology shock; they are equivalent to total factor productivity.

2. The *labor wedge* $\Delta_{lt} = (1-\tau_{lt})$ captures all the disturbances in the labor market as well as the distortions in other sectors that have an impact on the labor market. It captures the discrepancy between the intra-temporal marginal rate of substitution of leisure to consumption and the marginal product of labor.

3. The *investment wedge* $\Delta_{xt} = (1/(1+\tau_{xt}))$ captures the distortions in the capital market. They represent the discrepancy between the inter-temporal marginal rate of substitution and the return on investment.

4. The *government wedge* $\Delta_{gt} = g_t$ is the distortions in the resource constraint and corresponds to the government purchases in the data.

5. The *preference wedge* $\Delta_{ct} = (1/(1+\tau_{ct}))$ captures the discrepancy between the intra-temporal marginal rate of substitution of home consumption to foreign consumption and the relative price of those goods.

6. The *asset wedge* $\Delta_{bt} = (1-\tau_{bt})$ captures the distortions in the Euler equation and represents the discrepancy between the inter-temporal marginal rate of substitution and the return on foreign asset.

Notice that one could consider other models where we change the location of the wedges. But, if all the possibilities are considered, those models would capture the same features present in the model I previously described. For example, we could add a wedge on capital, but this would capture the same distortion as the investment wedge.

Definition of the equilibrium.– A competitive equilibrium of the prototype economy consists of wedges $\{A_t, \tau_{lt}, \tau_{xt}, g_t, \tau_{ct}, \tau_{bt}\}$, allocation $\{c_{ht}, c_{ft}, l_t, k_t, b_t\}$, and prices $\{r_t, r_t^*, w_t, p_t\}$, for each country, such as:

i- Given the wedge A_t and the prices $\{r_t, w_t, p_t\}$, the firm of each country maximizes its profit,

ii- Given the wedges $\{\tau_{lt}, \tau_{xt}, \tau_{ct}, \tau_{bt}\}$ and the prices $\{r_t, r_t^*, w_t, p_t\}$, the household of each country maximizes its life-time utility,

iii- the government budget constraint is satisfied for each country (equation 7),

iv- the resource constraint for each country holds,

$$c_{ht}^i + c_{ft}^j + x_t^i + g_t^i = y_t^i \tag{8}$$

v- the international asset market clears (equation 6).

Remark 1. From the description of the household environment, the consumers of country i can invest in local capital with a net rate of return r^i or/and participate in the international financial market with a return of $r*$. We expect, without non-arbitrage conditions, that the household invests only in the asset with the higher return such that the local investment and the foreign asset are redundant. However, the two assets are not redundant in my setting. They play different roles. The local investment ensures the building up of the capital necessary for the final good production in each country. Thus, as an investment in the capital comes only from the home household, it cannot be null every period. Foreign asset plays the role of international finance as a financial counterpart of the trade in good. Then, if a country faces a bad shock, it can borrow from abroad, and in good times, it can pay back the debt; it is international insurance. The following proposition describes the role of foreign assets in our prototype economy.

Proposition 1. *When the foreign asset market is nonexistent, i.e., $b_t^h = b_t^f = 0$, in equilibrium, trade is balanced each period, and there is a lack of international finance. In our framework, there exists a period during which a country incurs debt from abroad.*

Proof.– (See Appendix A.1) It comes out from the proof that without the international financial market, the net export of each country is zero every period. The rationale behind the proposition is that, as the two economies are not identical in terms of the size of the population, the production process, and preference for foreign goods, it is unusual that the exports offset each period the imports. Thus, the international financial market exists.

3 The accounting procedure

The accounting procedure consists first in measuring the different wedges, and second in evaluating the contribution of those wedges (one at a time or in combination).

Measuring the wedges.– For this step, we use data on aggregate variables and compute the different wedges using the equations derived from the prototype equilibrium conditions. The wedges are then measured using the following equations from the equilibrium conditions $\forall i = h, f$:

$$c^i_{ht} + c^j_{ft} + x^i_t + g^i_t = y^i_t \tag{9}$$

$$y^i_t = A^i_t F(k^i_t, (1+\gamma)^t l^i_t) \tag{10}$$

$$u^i_{c_f t}(.) = u^i_{c_h t}(.)(1+\tau^i_{ct})\frac{p^j_t}{p^i_t} \tag{11}$$

$$u^i_{lt}(.) = -u^i_{c_h t}(.)(1-\tau^i_{lt})(1+\gamma)^t F^i_{lt} \tag{12}$$

$$u^i_{c_h t}(.)(1+\tau^i_{xt}) = \beta^i E_t\left[u^i_{c_h t+1}(.)(F^i_{kt+1} + (1-\delta)(1+\tau^i_{xt+1}))\right] \tag{13}$$

$$E_t\left[1 + (1-\tau^i_{bt+1})r^*_{t+1}\right] = \frac{1}{1+\tau^i_{xt}} E_t\left[\frac{p^i_{t+1}}{p^i_t}\left(F_{1t+1} + (1-\delta)(1+\tau^i_{xt+1})\right)\right], \tag{14}$$

where H_{zt} denotes the derivative of the function with respect to its argument z.

Measuring the contribution of wedges.– The measurement of the contribution of wedges consists in using my prototype model to perform some counterfactual analysis. For that purpose, I conduct different experiments to isolate the effect of wedges. In other words, I make some wedges fluctuate and shut down the fluctuation of the remaining wedges by setting their values to a constant. For example, to evaluate the contribution of the country i efficiency wedge, we make this wedge (A^i_t) fluctuate and set the other wedges constants ($\forall t, \omega_t = \omega_1$, where ω stands for all of the other wedges in the model). After, the goal now is to solve the model to back up the aggregate variables (the allocation of the economy). Those variables represent then how the economy would have evolved if the only distortion in the economy express as the total productivity shock. Notice that the allocation back up as well as the associated prices must satisfy the competitive equilibrium of the economy. The proposition states the conditions of an equilibrium allocation.

Proposition 2. *Given the wedges* $\{\tau^i_{ct}, \tau^i_{lt}, \tau^i_{bt}, \tau^i_{xt}\}, i = h, f$, *a competitive equilibrium allocation of the economy solves the following equation (15) to the equation (22)*

$$u^i_{lt}(.) = -u^i_{c_h t}(.)(1-\tau^i_{lt})F^i_{c_f t}(.) \tag{15}$$

$$u^j_{lt}(.) = -u^j_{c_h t}(.)(1-\tau^i_{lt})F^j_{c_f t}(.) \tag{16}$$

$$\frac{u^i_{c_f t}(.)}{u^i_{c_h t}(.)(1+\tau^i_{ct})} = \frac{u^j_{c_h t}(.)(1+\tau^j_{ct})}{u^j_{c_f t}(.)} \tag{17}$$

$$c^i_{ht} + c^j_{ft} + k^i_{t+1} + g^i_t = F(k^i_t, z^i_t l^i_t) + (1-\delta)k^i_t \tag{18}$$

$$c^j_{ht} + c^i_{ft} + k^j_{t+1} + g^j_t = F(k^j_t, z^j_t l^j_t) + (1-\delta)k^j_t \tag{19}$$

$$u^i_{c_h t}(.)(1+\tau^i_{xt}) = \beta E_t\left[u^i_{c_h t+1}(.)\left(F^i_{c_h t+1}(.) + (1-\delta)(1+\tau^i_{xt+1})\right)\right] \tag{20}$$

$$u^j_{c_h t}(.)(1+\tau^j_{xt}) = \beta E_t\left[u^j_{c_h t+1}(.)\left(F^j_{c_h t+1}(.) + (1-\delta)(1+\tau^j_{xt+1})\right)\right] \tag{21}$$

$$E_t\left[\frac{1}{(1-\tau^i_{bt+1})}\frac{1}{1+\tau^i_{xt}}(F^i_{kt+1} + (1-\delta)(1+\tau^i_{xt+1})) - 1)\right] = E_t\Big\{\frac{1}{(1-\tau^j_{bt+1})} \tag{22}$$
$$(\frac{1}{1+\tau^j_{xt}}\frac{u^i_{c_f t+1}(.)}{u^i_{c_h t+1}(.)(1+\tau^i_{ct+1})}\frac{u^i_{c_h t}(.)(1+\tau^i_{ct})}{u^i_{c_f t}(.)}(F^j_{kt+1} + (1-\delta)(1+\tau^j_{xt+1})) - 1)\Big\}$$

Proof.– (See appendix A.2)
The procedure of the proof is to recover the prices from the allocation that satisfies the equation (15) to the equation (22). Then show that given those prices, households, and firms optimize in each country, the resource constraints are verified and all markets are cleared.

4 Quantitative Analysis: application of the accounting procedure to the US and Canada

This section provides the procedure of application of the international business cycle accounting to the US and Canada. The goal is to account for the business cycle of the US and Canada during the great recession of 2007-2008. The findings I will present in this section are based on the assumption that agents have perfect foresight. This implies that they have accurate and complete information about future economic conditions such that they face no uncertainty.

4.1 Calibration procedure

For the application, I use common functional forms in business cycle literature. I opt for a Cobb-Douglas form $F(k,l) = k^{\alpha}l^{1-\alpha}$ for the production function and for the utility function the form $U(c,l) = \log(c) + \psi\log(1-l)$ with an Armington aggregation for the consumption $\left(c_h^{\frac{\sigma-1}{\sigma}} + c_f^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. The parameters I use are also familiar to business cycle literature. I choose the capital share α as one-third and the time allocation parameter $\psi = 2.5$. The Armington aggregator coefficient I use in the benchmark model is 2 for the two countries. I then vary those coefficients for the robustness check. I choose the depreciation rate δ and the discount factor β so that, on an annualized basis, depreciation is 5% and the rate of time preference 2.5%. I use the data to compute country-specific growth of population and the rate of labor-augmenting technical progress.

Only the aggregate data are needed for the application of the method. I collect those data for the US and Canada using the OECD database. We need, for each country, the output, the labor, the investment, the government consumption, the private consumption, the world import and export, and the bilateral import between the US and Canada. The model distinguishes home consumption from foreign consumption in opposite to what we have in data. To overcome this issue, I consider the total import from the partner country as the foreign consumption good. Then, the home consumption good is the aggregate consumption good minus the foreign consumption good. As the world economy consists of more than two countries and each country trades not only with each other, we let the government wedge be the net export of all other trade partners except Canada and the US in the data. Following the same rationale as Chari, Kehoe, and McGrattan (2004), the government wedges capture the relationship between, each country and the rest of the world. So the government wedges represent the net export with the rest of the world. The data I use to measure the wedge and the counterfactual experiments are quarterly data for 2000:1 to 2014:4. As I mentioned, the data mainly come from the OECD database. However, in order to estimate the share of foreign consumption goods in the aggregate consumption goods, I use the import of the counterpart partner country from the DOTS (Direction Of Trade Statistics) database of the IMF (International Monetary Fund).

In order to reconcile the model and the data, I use per capita variables deflated by the GDP deflator. Indeed, the model is a representative agent model, thus using per capita variables in the model and data makes the approximation realistic.

4.2 Findings

In this section, I present the results of the accounting procedure performed for the US and Canada. I focus on describing the 2007-2008 crisis and the accounting of this business cycle.

Description of the crisis.– I begin by providing some descriptive statistics for the US and Canada during the period of the 2007-2008 crisis. The evolution of GDP, private consumption, investment, and hours worked for the US and Canada are described in Figure 1 and Figure 2 respectively. The decline of the aggregate variables during the recession started in the fourth quarter of 2007 and reached the through in the fourth quarter of 2009. In the US, the recession was characterized by a decline of output by about 4%, while investment and labor declined by about 21% and 9%, respectively, from the first quarter of 2008 to the third quarter of 2009 (see Table A.1). Concerning Canada, from the first quarter of 2008 to the third quarter of 2009, output fell by about 3%, investment fell by about 12%, and labor by about 6% (see Table A.1).

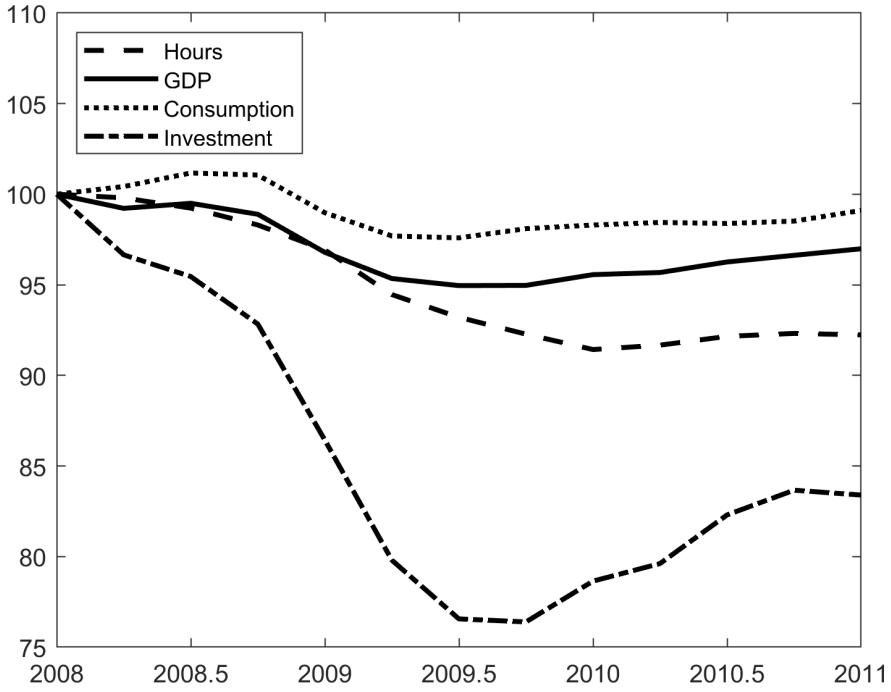


Figure 1: Description of the 2007-2008 crisis in the US

Notes.- The figure shows the evolution of hours worked, GDP, consumption, and investment for the US in percentages of their values in the first quarter of 2008. Source: OECD data and the author's calculations.

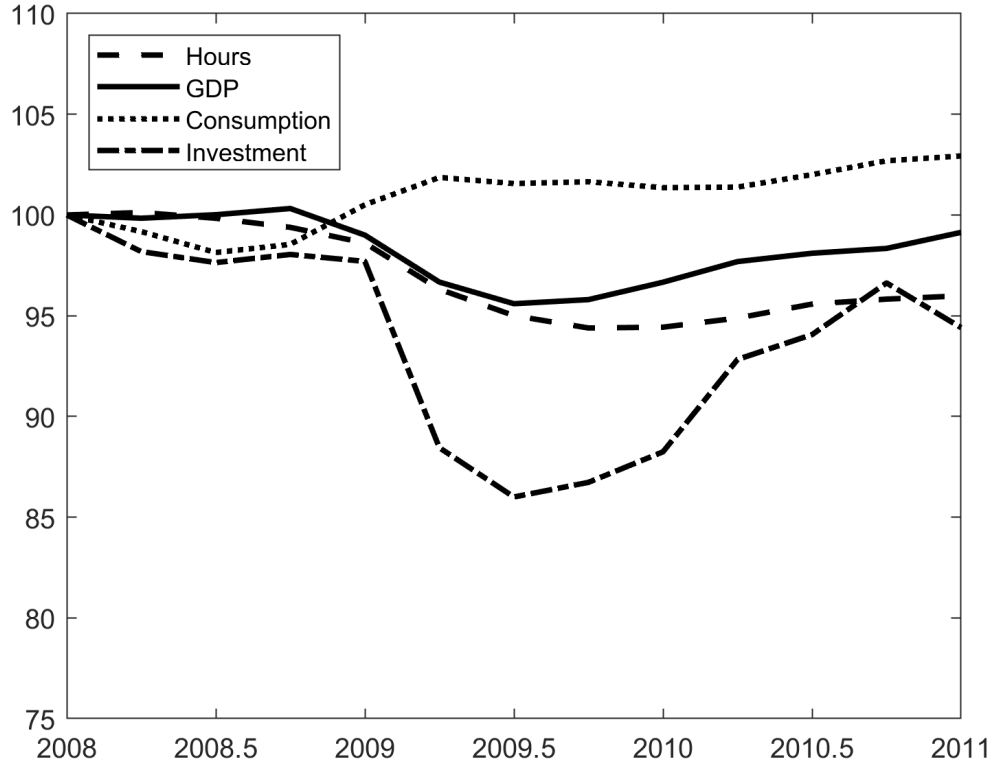


Figure 2: Description of the 2007-2008 crisis in Canada

Notes.- The figure shows the evolution of hours worked, GDP, consumption, and investment for Canada in percentages of their values in the first quarter of 2008. Source: OECD data and the author's calculations.

Wedges measurement.– I begin the analysis of the measured wedges by describing how they evolve during the period of the recession. The Table (1) reports the percentage changes in the wedges between the first quarter of 2008 to the third quarter of 2009. For Canada, during this period, they have been a drop in the investment wedge by about 7%, in the labor wedge by about 14%, and in the foreign asset wedge, dramatically, by about 77%. At the same time the efficiency wedge, the preference wedge, and the government wedge have increased respectively by 0.3%, 14%, and 8%. In the US, they have been also a decrease in the preference wedge by about 4%, in the labor wedge by about 9%, in the investment wedge by about 15%, and in the foreign asset wedge by about 16%. We also registered an increase in the US efficiency wedge and government wedge by 2% and 8.8% respectively. To further get insights on how the evolution of the wedges was associated with that of aggregate variables during the recession, for each country, I plot the wedges and some variables. Those graphs are presented in the appendix.

They give broad information on the comovement of wedges and the considered aggregate variable. For example, from Figure (A.1a) we could expect that the efficiency wedge and labor wedge of Canada play a role in the fluctuation of Canada's output during the recession. As not only the wedges in Canada, for example, can explain the fluctuations of aggregate variables in Canada we cannot surely draw a pattern from this analysis. I then, perform a counterfactual analysis to evaluate the contribution of some wedges.

Table 1: % variation of wedges from 2007:4 to 2009:4

	Δ_{at}	Δ_{lt}	Δ_{xt}	Δ_{gt}	Δ_{ct}	Δ_{bt}
Canada	0.32	-14.22	-7.24	8.01	14.38	-76.94
US	2.00	-9.11	-15.39	8.83	-3.79	-15.85

Notes.- The Table shows the variation of wedges in percentages for the US and Canada. Δ_{at} stands for efficiency wedge, Δ_{lt} labor wedge, Δ_{xt} investment wedge, Δ_{gt} government wedge, Δ_{ct} preference wedge, and Δ_{bt} foreign asset wedge. Source: The author's calculations.

Evaluation of wedges' contribution .– To better assess the role played by disturbances in each market, I evaluate the contribution of each wedge in the fluctuation of aggregate variables during the recession. Using the approach described in paragraph (3) of the section (3), I assess how the aggregate variables would have fluctuated considering, once at a time, the fluctuation of each wedge. In other words, I determine how output would have fluctuated if the only distortion was the efficiency wedge. Considering the output, I do the same exercise, as for the efficiency wedge, with the remaining wedges in both countries. To summarize the contribution of each wedge, I rely on the ϕ statistic proposed by Brinca et al. (2016).

The ϕ statistic is the inverse of the mean-square error of each wedge. It captures how close a simulated variable (variable obtained from simulation when assumed that only some wedges fluctuate) is to its equivalent in the data. Let S_t be one of the aggregate variables in data, S_{mt} be the counterpart from the simulation of S_t assuming that the only wedge that fluctuates is m. The ϕ_m^S statistic, measuring the contribution of the wedge m in the fluctuation of the variable S is:

$$\phi_m^S = \frac{1/\sum_t \left(S_t - S_{mt}\right)^2}{\sum_j \left[1/\sum_t \left(S_t - S_{jt}\right)^2\right]}, \tag{23}$$

where $(m, j) \in \left\{\Delta_{at}^i, \Delta_{lt}^i, \Delta_{xt}^i, \Delta_{gt}^i, \Delta_{pt}^i, \Delta_{bt}^i\right\}_{(i \in \{Ca, US\}}$.

The ϕ statistic lies in $[0, 1]$ and sums to 1 for all the twelve wedges in both countries for each variable. The more the statistic is close to one, the more the wedge contributes to the fluctuation of the variable. Thus, when the simulated output and its counterpart in data fit perfectly (meaning $y_t - ymt = 0$ for all t), then $\phi_m^Y = 1$. When a wedge does not contribute to the fluctuation of a variable, the ϕ statistic is near zero.

Table 2 summarizes the ϕ statistics computed for both the US and Canada during the recession from the fourth quarter of 2007 to the fourth quarter of 2009. The results, shown in the table, concern the counterfactual analysis when we feedback the wedges one at a time to the model. for the counterfactual experiments we conducted. Let's first focus on the analysis of the contribution of the wedges to the fluctuation of output in the US and Canada. The accounting procedure reveals that the output drop in Canada during the recession was caused mainly by discrepancies in the efficiency wedge and the labor wedge in the US. Those wedges account for about 26% and 14% respectively in the decline of the output in Canada. In other words, the distortions that caused disturbances in the labor wedge and the productivity in the US explain the main drop in the output of Canada. However, the drop in output in the US was mainly due to the discrepancies in the investment wedge and the labor wedge in the US. They respectively account for the 27% and 16% of the output drop in the US. The investment wedge and the preference wedge in Canada contribute by about 11% and 10% to the decline of the US output. To sum up, the output decline in the US and Canada during the recession was caused mainly by the efficiency wedge, the labor wedge, and the investment wedge in the US. A second role can be attributed to the investment wedge and preference wedge in Canada.

Concerning the decline of investment, in the US and Canada, during the crisis the primary role is attributed to the discrepancies in investment wedges both in the US and Canada. They account respectively by 20% and 40% for the decline of investment in Canada, and respectively by 29% and 9% in the decline of investment in the US. Compared with the business cycle accounting of the output, we notice that the investment in the US and Canada are more affected by the shocks in both countries.

Finally, the employment business cycle accounting for the US and Canada reveals again that the main forces are the discrepancies in the efficiency wedge, labor wedge, and investment wedge in the US and the discrepancies in the investment wedge in Canada that explain mainly the drop of labor. With respectively 19% and 13% the efficiency wedge and labor wedge in the US explain the drop of employment in Canada. While the decline of employment in the US is due to about 27%

of the discrepancies in the investment wedge in Canada and respectively 25% and 19% of discrepancies in the investment wedge and labor wedge in the US.

The takeaway of the counterfactual analysis is that the US and Canada were affected by the distortions in each other economies during the crisis. However, the disturbances in the US economy seem to have impacted more the economy of Canada. The labor wedge and efficiency wedge in the US played the most important role in Canada during the recession, with a non-negligible role for the US investment wedge and Canada investment wedge in the fluctuation of Canada's investment. Concerning the fluctuations in aggregate variables in the US, the most important role comes from the investment wedge, the labor wedge, and the efficiency wedge in the US. Nevertheless, the main role in the decline of labor in the US was due to the investment wedge in Canada.

5 Discussions

The quantitative results I presented suppose that the agents in the economy don't face future economic uncertainty. I assume a perfect foresight economy where the agents have complete and accurate information. We must have this assumption in mind when interpreting the results. Indeed, the assumption of perfect foresight impacts the decision of agents concerning the investment in capital and their participation in the international financial market. Thus the two Euler equations of the equilibrium would have been impacted in the quantitative experiments.

However, my results indicate that distortions in the US economy, especially in the labor wedge, the efficiency wedge, and the investment wedge in the US, have a significant impact on the Canadian economy. Similarly, distortions in the Canadian economy, specifically those affecting the investment wedge, have an impact on the US economy. According to the number of distorted markets in the US explaining the fluctuation of the aggregate variables in both countries, we may conclude that the 2007-2008 crisis probably originated in the US and then spread to Canada.

In addition, as we mentioned in the introduction, Chari et al. demonstrated that an open economy is equivalent to a prototype closed economy with a government wedge. So what differentiates the Business Cycle Accounting proposed by Chari et al. (2007) for a closed economy from ours? To answer this question, we compare the ϕ statistics from our study and those obtained from Brinca et al. (2016) for output. The results presented in Table 3 indicate that a particular country's business cycle is mainly due to distortions in that country, and the impact of foreign countries is through the government wedge. For example, Chari et al.'s accounting procedure attributes most of the fluctuations of output in Canada to the

efficiency wedge and the investment wedge in Canada, while my accounting procedure attributes the same output movements to the labor wedge and efficiency wedge in the US. Thus, ignoring the role of distortions to the US economy in the business cycle of Canada, and vice-versa, could misleading policies. This analysis shows that the International Business Cycle Accounting methodology highlights the interdependence between countries.

Table 2: ϕ statistics in % for 2007:4 to 2009:4

	Canda Wedges						US Wedges					
	Δ_{at}	Δ_{lt}	Δ_{xt}	Δ_{gt}	Δ_{ct}	Δ_{bt}	Δ_{at}	Δ_{lt}	Δ_{xt}	Δ_{gt}	Δ_{ct}	Δ_{bt}
Y Ca	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
X Ca	1.70	3.80	40.02	1.26	2.16	3.72	12.64	4.32	20.48	1.36	4.73	3.80
X US	6.79	8.94	8.36	9.17	5.73	5.97	4.09	2.30	28.96	1.57	9.42	8.70
L Ca	0.76	2.01	10.90	1.13	5.44	7.24	18.52	12.99	5.33	21.33	10.64	3.71
L US	2.70	0.34	26.50	1.89	0.46	8.48	1.80	18.79	25.58	4.00	5.09	4.38

Notes.- The Table shows the contribution in percentages of each wedge in the fluctuation of each country output Y, investment X, and labor L. Δ_{at} stands for efficiency wedge, Δ_{lt} labor wedge, Δ_{xt} investment wedge, Δ_{gt} government wedge, Δ_{ct} preference wedge, and Δ_{bt} foreign asset wedge. Source: The author's calculations.

Table 3: ϕ statistics comparison for IBCA and BCA

		Δ_{at}	Δ_{lt}	Δ_{xt}	Δ_{gt}	Δ_{ct}	Δ_{bt}	Δ_{at}	Δ_{lt}	Δ_{xt}	Δ_{gt}	Δ_{ct}	Δ_{bt}
IBCA	**Y Ca**	0.14	0.72	4.98	0.71	6.37	1.57	13.92	26.37	8.25	23.25	6.52	7.20
	Y US	1.52	4.87	10.50	1.70	10.28	5.42	2.97	15.69	27.02	5.80	4.82	9.41
BCA	**Y Ca**	49.00	13.00	18.00	20.00	-	-	-	-	-	-	-	-
	Y US	-	-	-	-	-	-	16.00	46.00	32.00	6.00	-	-

Notes.- The Table shows the contribution in percentages of each wedge in the fluctuation of each country output Y, investment X, and labor L. Δ_{at} stands for efficiency wedge, Δ_{lt} labor wedge, Δ_{xt} investment wedge, Δ_{gt} government wedge, Δ_{ct} preference wedge, and Δ_{bt} foreign asset wedge. IBCA is International Business Cycle Accounting Method and BCA is Business Cycle Accounting Method. Source: The author's calculations.

6 Conclusion and extensions

In this paper, I propose a method that can provide insights for researchers to better specify their quantitative models in international business cycle studies. The method focuses on an accounting procedure based on a prototype model of international growth that includes wedges to capture potential frictions and distortions in markets. For each country, I include an efficiency wedge, a labor wedge, an investment wedge, a government wedge, a preference wedge, and a foreign asset wedge. The evaluation of the contribution of the wedges to the fluctuation of aggregate variables provides insight into which frictions generate the business cycle and the comovement observed in the data. Theoretically, business cycle comovements across countries are allowed through the trade in goods and the international financial market present in this model.

To demonstrate the method, I apply it to study the synchronization of the business cycle during the great recession in the US and Canada. My results show, during this period, the primary role in the economic downturns in both countries is attributed to the disturbances in the labor market, investment, and in productivity in the US. However, the disturbances in investment in Canada play a second role in the fluctuation of aggregate variables in both countries. These results suggest that Canada and the US are heavily linked. This is consistent with the high trade as well as the financial transactions between the two countries. The results suggest also that the 2007-2008 crisis probably originated in the US and then spread to Canada.

A further step for more accuracy of the results would be to add uncertainty in the agents' decisions in my quantitative analysis. Adding the uncertainty could affect the measurement of the wedges so the counterfactual analysis. Indeed, the uncertainty affects the agents through their investment decision in local capital and foreign assets.

To classify the details model in international macroeconomics in terms of theoretical extra work, we need to establish their equivalence to the prototype model by introducing specific wedges. However, we still need to address the identification problem in inferring the stochastic distribution of two countries and twelve exogenous variables. Further thought is required in this area.

One direction for this work could be to explore the puzzle literature in international macroeconomics, such as the trade co-movement puzzle. To do this, we would need to apply our methodology by developing a detailed model based on the insights gleaned from the accounting procedure. Then, we can examine whether the model can help resolve the puzzle.

References

Ambler, S., Cardia, E., & Zimmermann, C. (2004). International business cycles: What are the facts? *Journal of monetary economics*, *51*(2), 257–276.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International real business cycles. *Journal of political Economy*, *100*(4), 745–775.

Bems, R., Johnson, R. C., & Yi, K.-M. (2010). Demand spillovers and the collapse of trade in the global recession. *IMF Economic review*, *58*(2), 295–326.

Bordo, M. D. (2012). *The great depression and the great recession: what have we learnt?*

Brinca, P., Chari, V., Kehoe, P., & McGrattan, E. (2016). Accounting for business cycles. In *Handbook of macroeconomics, 2016* (pp. 1013–1063). Elsevier BV.

Chari, V. V., Kehoe, P. J., & McGrattan, E. (2004). *Business cycle accounting.* National Bureau of Economic Research Cambridge, Mass., USA.

Chari, V. V., Kehoe, P. J., & McGrattan, E. R. (2007). Business cycle accounting. *Econometrica*, *75*(3), 781–836.

Eaton, J., Kortum, S., Neiman, B., & Romalis, J. (2016). Trade and the global recession. *American Economic Review*, *106*(11), 3401–3438.

Gao, X. (2007). Business cycle accounting for the chinese economy. ..

Heathcote, J., & Perri, F. (2002). Financial autarky and international business cycles. *Journal of monetary Economics*, *49*(3), 601–627.

Huo, Z., Levchenko, A. A., & Pandalai-Nayar, N. (2023). Utilization-adjusted tfp across countries: Measurement and implications for international comovement. *Journal of International Economics*, 103753.

Johnson, R. C. (2014). Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics*, *6*(4), 39–83.

Kobayashi, K., & Inaba, M. (2006). Business cycle accounting for the japanese economy. *Japan and the World Economy*, *18*(4), 418–440.

Kose, M. A., Otrok, C., & Whiteman, C. H. (2003). International business cycles: World, region, and country-specific factors. *american economic review*, *93*(4), 1216–1239.

Kose, M. A., & Yi, K.-M. (2001). International trade and business cycles: is vertical specialization the missing link? *American Economic Review*, *91*(2), 371–375.

Kose, M. A., & Yi, K.-M. (2006). Can the standard international business cycle model explain the relation between trade and comovement? *Journal of international Economics*, *68*(2), 267–295.

Lama, R. (2005). *Business cycle accounting in developing countries.*

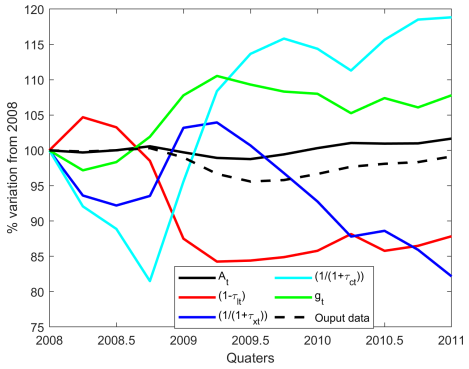
Obstfeld, M., & Rogoff, K. (2000). The six major puzzles in international macroeconomics: is there a common cause? *NBER macroeconomics annual*, *15*, 339–390.

Otsu, K. (2009, November). *International Business Cycle Accounting* (IMES Discussion Paper Series No. 09-E-29). Institute for Monetary and Economic Studies, Bank of Japan. Retrieved from https://ideas.repec.org/p/ime/imedps/09-e-29.html

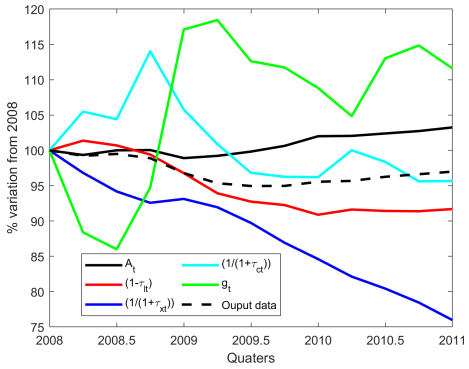
Stockman, A. C., & Tesar, L. L. (1995). Tastes and technology in a two-country model of the business cycle: explaining international comovements. *The American Economic Review*, *85*(1), 168–185.

Wen, Y. (2007). By force of demand: Explaining international comovements. *Journal of Economic Dynamics and Control*, *31*(1), 1–23.

A Evolution of wedges and aggregate variables

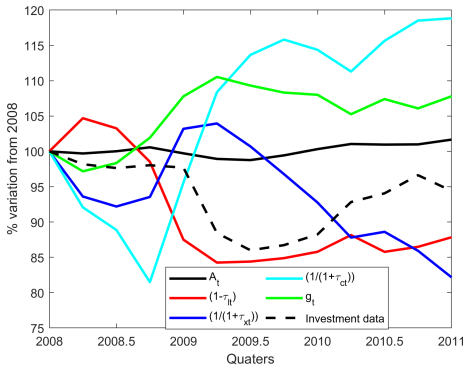


(a) Output and wedges for Canada

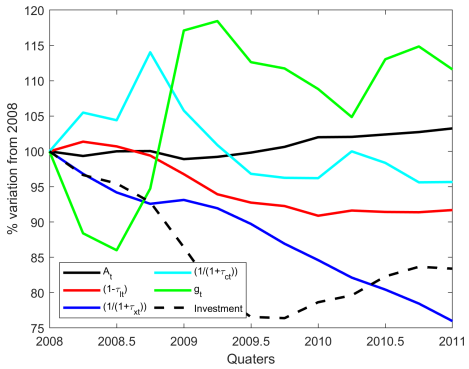


(b) Output and wedges for US

Figure A.1: Output and wedges

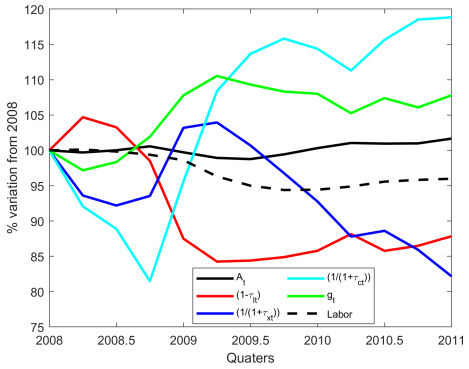


(a) Investment and wedges for Canada

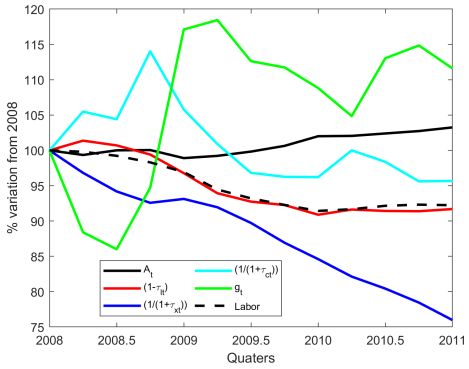


(b) Investment and wedges for US

Figure A.2: Investment and wedges



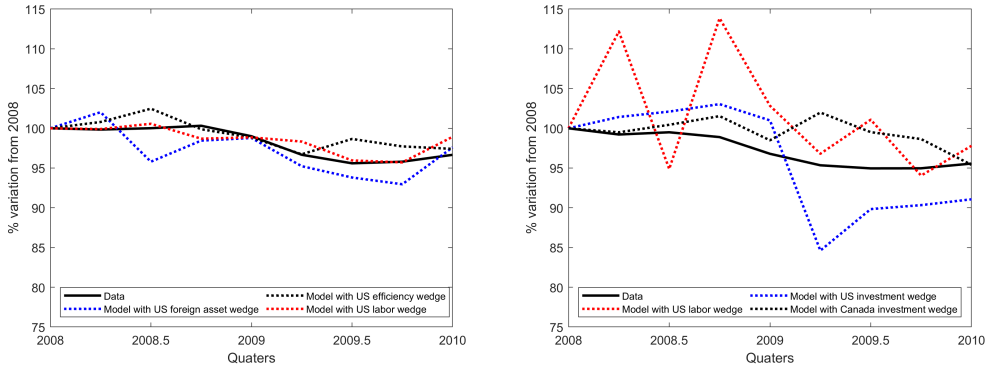
(a) Labor and wedges for Canada



(b) Labor and wedges for US

Figure A.3: Labor and wedges

B Output Data and Output Prediction of models with just one wedge



(a) Output for Canada (b) Output for US

Figure B.1: Output in Data and Output models predictions

A 2007-2008 crisis Description

Table A.1: Changing in aggregates during the crisis

			Change of aggregates in %		
	Peak	Through	Y	X	L
Canada	2007Q4	2009Q4	-3.34	-11.76	-5.57
US			-4.44	-21.35	-8.57

Notes.- The Table shows the decrease in percentages of output Y, investment X, and labor L and the periods of the peak and Through of those variables for the US and Canada. Q4 stands for the fourth quarter. Source: The author's calculations.

A Proof of propositions

A.1 Proof of proposition 1

Without assets trading in the world economy, the net export equals zero every period. We know that the net export is:

$$(X-M)_t^i = p_t^i(y_t^i - c_{ht}^i - x_t^i) - p_t^j c_{ft}^i$$

As the firms are in a competitive market, their profit is zero every period such that

$$p_t^i y_t^i = w_t^i l_t^i + r_t^i k_t^i$$

Then by replacing the firm revenue with the net export equation, we get:

$$(X-M)_t^i = w_t^i l_t^i + r_t^i k_t^i - p_t^i(c_{ht}^i + x_t^i) - p_t^j c_{ft}^i$$

We also know that the government wedge is equal to the transfers to the household state by the following equation:

$$tr_t^i = g_t^i = p_t^j \tau_{ct}^i c_{ft}^i + p_t^i \tau_{xt}^i x_t^i + \tau_{lt}^i w_t^i l_t^i \tag{24}$$

Then using the budget constraint 4 (with $b_t^i = 0$) and 24, we obtain $(X-M)_t^i = 0$.

A.2 Proof of proposition 2

Let assume $\mathcal{A}^i = \left(c_{ht}^i, c_{ft}^i, x_t^i, l_t^i, b_t^i\right)_{t=0}^{\infty}$ i=(h,f) solve equations in proposition 2 and let prove there exist price $\mathcal{P}^i = (p_t^i, w_t^i, r_t^i, r_t^*)_{t=0}^{\infty}$ such that $\mathcal{A}^i$ and $\mathcal{P}^i$ is a competitive equilibrium.
The proof is straightforward when we make the following assumptions:

- Normalize a price: $\forall t, p_t^h = 1$
- Compute $p_t^f = \frac{u_{2t}^h(.)}{u_{1t}^h(.)(1+\tau_{ct}^h)}$
- Compute $w_t^i = p_t^i F_{2t}^i(.)$
- Compute $r_t^i = p_t^i F_{kt}(.)$
- Using non arbitrage condition, compute $r_{t+1}^* = \frac{1}{(1-\tau_{bt+1}^i)}(\frac{1}{1+\tau_{xt}^i}\frac{p_{t+1}^i}{p_t^i}(F_{kt+1} + (1-\delta)(1+\tau_{xt+1}^i)) - 1)$

Indeed,

- From the wage rate and interest rate equations, firms optimize as shown by equations 25 and 26;
- From equations 18 and 19 of proposition 2, resource constraints are satisfied;
- Combining the remind equations and prices we get the FOC of households problems (equations 27 to 30 for each country i) ;
- The budget constraints (equations 4 for each country) are satisfied by using them to compute the assets variables.

First order conditions of firms optimization problems

$$w_t^i = p_t^i F_{lt}(.) \tag{25}$$

$$r_t^i = p_t^i F_{kt}(.) \tag{26}$$

First order conditions of household optimization problems

Using the Lagrangian procedure we get the following equations:

$$u_{c_f t}(.) = u_{c_h t}(.)(1 + \tau_{ct}^i)\frac{p_t^j}{p_t^i} \tag{27}$$

$$u_{lt}(.) = -u_{c_h t}(.)(1 - \tau_{lt}^i)\frac{w_t^i}{p_t^i} \tag{28}$$

$$u_{c_h t}(.)\frac{1}{p_t^i} = \beta E_t \left[u_{c_h t+1}(.)(1 + (1 - \tau_{bt+1}^i) r_{t+1}^*)) \frac{1}{p_{t+1}^i} \right] \tag{29}$$

$$u_{c_h t}(.)(1 + \tau_{xt}^i) = \beta E_t \left[u_{c_h t+1}(.)(\frac{r_{t+1}^i}{p_{t+1}^i} + (1 - \delta)(1 + \tau_{xt+1}^i)) \right] \tag{30}$$